WAVES IN A LIQUID WITH GAS BUBBLES
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It is well known that a liquid containing gas bubbles is a sharply nonlinear and dispersive medium. One of the most interesting wave states in such media is the propagation of solitary waves (solitons) [1, 2]. They are formed owing to reradiation by the bubbles of the energy stored in the pressure wave in the process of compression. When the amplitudes of the waves are sufficiently large, however, breakup of the bubbles occurs [3-5]. This should apparently strongly affect the formation and structure of waves in a liquid with gas bubbles. In [3-5] several mechanisms of breakup of single bubbles in shock waves are described. For example, a bubble can be broken up by the cumulative liquid jet that is formed when the bubble interacts with the wave [3]. In [4,5] a mechanism is proposed for bubble breakup as a result of the development of instability (Kelvin-Helmholtz) owing to the translational motion of the bubble in the pressure wave. Bubble break-up can also occur as a result of the development of Taylor instability, i.e., as a result of accelerated motion of the bubble-liquid interface in the wave [2]. Thus the breakup of single bubbles in a pressure wave has been studied quite well in the literature but the effect of the fragmented bubbles on the structure of the wave in a gas-liquid medium has practically not been studied.

In this work the behavior of gas bubbles in propagating solitary waves in aliquid containing gas bubbles was studed experimentally for different parameters of the waves and the medium. The effect of bubble fragmentation on the structure of the wave was also studied.

The experiments were performed in a "shock tube" apparatus [6]. The working section was filled with liquid and saturated with gas bubbles through a bubble generator. A bell-shaped pressure wavewas generated by impact of a piston against the bottom of a liquid-filled transitional chamber and then propagated into the working section. The profiles of the pressure waves were recorded with piezoelectric pressuregauges, arranged along the working section and flush-mounted toits inner walls. The signals from the pressure gauges were fed into cathode-ray oscillographs. Filming of the gas bubbles, using a VSK-5 motion picture camera, with a frame speed of $50-250$ thousand frames/sec was performed in an optical insert in the working section synchronously with the passage of the wave. The parameters of the waves and the medium were varied as follows: the wave intensity ( $5 \leq \Delta \mathrm{p} / \mathrm{p}_{0} \leq 100$ ), the viscosity $4 \cdot 10^{-6} \mathrm{~m}^{2} / \mathrm{sec} \leq v \leq 8 \cdot 10^{-5} \mathrm{~m}^{2} / \mathrm{sec}$ ), the surface tension of the liquid ( $30 \cdot 10^{-3} \mathrm{~N} / \mathrm{m} \leq \sigma \leq 7 \cdot 10^{-2}$ $\mathrm{N} / \mathrm{m})$, the bubbie size $\left(1 \mathrm{~mm} \leq \mathrm{R}_{0} \leq 2 \mathrm{~mm}\right)$, and the asphericity of the bubbles $(1 \leq \alpha \leq 2)$ ( $\alpha=a / b$, where $a$ and $b$ are the long and short semiaxes of an ellipsoidal bubble). The gas content and the density of the liquid remained constant and equal to, respectively, $40=3 \cdot 10^{-3}$ and $\rho_{I}=1.04 \cdot 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$.

Figure 1 shows an oscillogram of the profile of a moderate-intensity pressure wave in a liquid with spherical bubbles of freon with radius $R_{0}=2 \mathrm{~mm}$ in an $80 \%$ solution of glycerine in water at a distance $x=0.32 \mathrm{~m}$ from the inlet into the gas-liquid medium (a) with $p_{0}=0.1$ MPa, $v=76.7 \cdot 10^{-6} \mathrm{~m}^{2} / \mathrm{sec}$, and $\sigma=0.068 \mathrm{~N} / \mathrm{m}$, as well as motion-picture frames of the behavior of a gas bubble at the times designated by the numbers on the oscillogram (b). At the stage of bubble compression the presence of a pressure gradient in the wave results in deformation of the leading edge of the bubble (Fig. 1b, 3; the arrow marks the direction of propagation of the wave). This is followed by the formation of a frontal cumulative jet, analogous to that described in [3] (Fig. 1b, 6). The jet penetrates through the bubble and drags with a it some of the gas into the surrounding liquid (Fig. 1b, 9). The high viscosity of the liquid stabilizes this process, and the part of the gas entrained by the jet is a single whole

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Fig. 1

bubble (Fig. 1b, 9-13). Thus the formation of a cumulative jet results in the division of a bubble into two bubbles, strung on a thin jet of liquid. Such fragmentation, however, virtually never changes the structure of the wave in subsequent oscillations of the bubbles, since the radius of the bubbles does not change significantly on breakup.

The velocity of the accumulative jet increases as the amplitude of the solitary pressure wave increases, and correspondingly a larger mass of the gas is carried by the jet into surrounding liquid. For intensities of the pressure wave $\Delta \mathrm{p} / \mathrm{p}>50$ the jet carries away all of the gas in the bubble. Figure 2 shows that the starting bubble is "transferred" by the jet into the surrounding liquid and in the process the bubble remains whole (Fig. $2 \mathrm{~b}, 6$ ), and the bubble continues to form together with the other bubbles a solitary pressure wave (Fig. 2a, 11-14).

Qualitatively different behavior of the bubble in a solitary wave is observed in a liquid filled with ellipsoidal gas bubbles. Figure 3 b shows a motion picture of the behavior of a nitrogen bubble in a pressure wave (Fig. 8a) : $R_{0}=1.5 \mathrm{~mm}, \mathrm{p}_{0}=0.1 \mathrm{MPa}, \alpha=1.3, \nu=15 \times$ $10^{6} \mathrm{~m}^{2} / \mathrm{sec}, \sigma=0.068 \mathrm{~N} / \mathrm{m}$. The bubble is overcompressed in the direction perpendicular to the propagating direction of the wave (Fig. 3b, 3-6). A ring-shaped jet of liquid, directed toward the center of the bubble, is formed. The appearance of the jet is caused by the asphericity of the bubble and the existence of a pressure gradient on its surface. An analogous ring-shaped jet of liquid, obtained in a numerical calculation of the collapse of an aspherical empty cavity at a solid wall, is described in [7]. When the ring-shaped jet closes two cumulative jets form: parallel and antiparallel to the direction of propagation of the


Fig. 4

wave (Fig. 3b, 7). The jet moving antiparallel to the wave is stopped almost immediately when it leaves the bubble (Fig. 3b, 8). The experiments showed that although the cumulative jet moves in the direction of the wave with a higher velocity in an aspherical bubble than in a spherical bubble with the same wave intensities, it carries into the surrounding liquid less gas than a jet in a spherical bubble. This is because the "transfer" of gas from the bubble occurs at a later stage of its expansion and the bubble has a larger volume (Fig. 3b, 7-10). As for spherical bubbles, however, such breakup does not significantly affect the structure of the wave. The fragmented bubbles continue to oscillate and form the next successive solitary pressure wave (Fig. 3b, 11-14).

It is well known that a bubble, having a radius $\mathrm{R}_{0} \approx(1-2) \mathrm{mm}$ and rising in a lowviscosity liquid, whirls about strongly, and in addition to being aspherical it is also strongly asymmetric. Motion-picture frames showing the breakup of sucha bubble in a pressure wave are presented in Fig. 4 ( $p_{0}=0.1 \mathrm{MPa}, \mathrm{R}_{0}=1.5 \mathrm{~mm}, \alpha=1.6, v=4 \cdot 10^{-5} \mathrm{~m}^{2} / \mathrm{sec}, \sigma=0.068 \mathrm{~N} / \mathrm{mm}$ ). As in Fig. 3, here the formation of a ring-shaped jet, but no longer uniformly over the surface of the bubble, is observed (Fig. 4b, 4). At the location where the curvature of the surface is greater the jet forms earlier and at each moment in time its velocity is higher than the part of the jet that formed on the opposite surface. In addition, owing to the asym metry of the bubble different parts of the ring-shaped jet move at different angles with respect to the propagating direction of the wave. This is the reason for the "acentral" closure of the ring-shaped jet, when in the course of breakup part of the jet does not participate in the formation of cumulative jets, passes by the location where the ring-shaped jet closes, and is directed toward the side wall of the bubble (Fig. 4b, 10). This part of the ring-shaped jet strikes the surface of the bubble and carriers off into the surrounding liquid part of the gas from the bubble (Fig. 4b, 11-12). Such lateral ejection of gas from the bubble, however, does not significantly affect the structure of the wave, since the size of the initial bubble does not change much.

Thus the experiments showed that in the process of jet breakup in pressure waves of moderate intensity the bubbles breakup into somewhat larger parts, which continue to oscillate


Fig. 6


Fig. 7
and form subsequent waves, and the effect of such breakup on the structure of a solitary wave is insignificant.

Figure 5 gives the results of measurments of the average velocity of a small cumulative jet of liquid in a spherical bubble $u$ as a function of the amplitude of the solitary wave $\Delta \mathrm{p} / \mathrm{p}_{0}$ in a wide range of values of the parameters of the medium: $\left.1-3\right) \mathrm{R}_{0}=2 ; 1$; 1 mm , $\sigma=0.068 ; 0.068 ; 0.035 \mathrm{~N} / \mathrm{m}, v=76.6 \cdot 10^{-6} ; 76.6 \cdot 10^{-6} ; 4 \cdot 10^{-6} \mathrm{~m}^{2} / \mathrm{sec}$, respectively. The velocity of the small cumulative jet was averaged in a time interval from the time at which it is formed up to the time at which it emerges from the bubble. The velocity of the small cumulative jet does not depend on the viscosity $v$ of the liquid, the surface tension $\sigma$, and the bubble sizes $\mathrm{R}_{0}$ in the range of parameters studied.

Figure 6 shows the dependence of the velocity $v$ of the small cumulative jet of liquid inside an aspherical bubble on the amplitude of the wave for different values of the starting deformation and parameters of the medium: $1-5$ ) $\mathrm{R}_{0}=1 ; 1.5 ; 1 ; 1 ; 1.5, v=4 \cdot 10^{-6} ; 15 \cdot 10^{-6}$; $4 \cdot 10^{-6} ; 14 \cdot 10^{-6} ; 4 \cdot 10^{-6} \mathrm{~N} / \mathrm{m}, \sigma=0.035 ; 0.068 ; 0.068 ; 0.035 ; 0.068 \mathrm{~N} / \mathrm{m}$, respectively. As for spherical bubbles, the velocity of the cumulative jet does not depend on the viscosity, the surface tension of the liquid, and the bubble size, but there is a significant dependence on the initial deformation $\alpha$. This is explained by the fact that for small initial deformations ( $\alpha \approx$ 1.2) a ring-shaped jet closes at a smaller angle relative to the propagating direction of the wave and most of the liquid passes into the cumulative jet, propagating parallel to the pressure wave.

From Figs. 5 and 6 one can see that the velocity of the cumulative jet is higher inside an aspherical bubble than inside a spherical bubble; this agrees with the computed data of [7].

In addition to the jet mechanism of bubble breakup, for sufficiently large wave amplitudes $\Delta \mathrm{p} / \mathrm{p}_{0} \geq 80$ there is observed experimentally another mechanism which is associated with the development of instability of the liquid-gas interface and breakup of a bubble into smaller gas inclusions. There exist two mechanisms of instability of the liquid-gas interface [4]. The Rayleigh-Taylor instability, which arises as a result of the acceleration of the interface, is characterized by a critical Bond number $B=\rho_{1} a R^{2} / \sigma$, where $\rho_{1}$ is the density of the liquid and a is the acceleration of the boundary of the liquid. The relative motion of the bubble in the liquid under the action of the pressure wave results in the formation of KelvinHelmholtz instability and the criterion characterizing this instability is the Weber number $W=\rho_{2} \Delta U^{2} R / 2 \sigma$ ( $\rho_{2}$ is the gas density and $\Delta U$ is the relative velocity of the bubble).


Fig. 8
Figure 7 b shows the behavior of a freon bubble with radius $\mathrm{R}_{0}=2 \mathrm{~mm}$ in a pressure wave having intensity $\Delta \mathrm{p} / \mathrm{p}_{0} \simeq 80$ (the wave profile is shown in Fig. 7a) with $\alpha=1.6, v=4 \cdot 10^{-6}$ $\mathrm{m}^{2} / \mathrm{sec}$, and $\sigma=0.068 \mathrm{~N} / \mathrm{m}$. The boundary of the bubble becomes unstable already in the first compression and waves cover the entire boundary (Fig. 7b, 4). A cumulative jet is not formed, and the bubble breaks up into very small parts and no longer undergoes oscillations (Fig. 7b, 9 and 10). As a result of this, a solitary wave is not formed from the initial pulse and the entire signal is covered with high-frequency fluctuations owing to oscillations of very small bubbles.

Figure 8 b shows motion-picture frames of the behavior of bubbles of a lighter gas (He) in a strong pressure wave (Fig. 3a). Compared with Fig. 7, under otherwise equal conditions, a cumulative jet forms in the He bubble and breakup into small parts in the first solitary wave is not observed (Fig. 8b, 4). Therefore the density of the gas in the bubble is the parameter that determines the breakup of the bubbles, i.e., in these experiments the KelvinHelmholtz mechanism of instability is realized. Bubble breakup by means of the KelvinHelmholtz instability is also observed in solitary waves at lower amplitudes, but at quite long times. Thus one can see in Fig. 2b (13) and Fig. 3b (11) that the boundary of the bubble is covered with disturbances which later result in bubble breakup. Therefore the high viscosity of the liquid prevents development of the Kelvin-Helmholtz instability while the presence of lateral ejections owing to the "acentral" closure of the ring-shaped jet accelerates this process.

## LITERATURE CITED

1. V. E. Nakoryakov, B. T. Pokusaev, and I. R. Shreiber, Wave Propagation in Gas- and VaporLiquid Media [in Russian], IT Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1983).
2. R. I. Nigmatulin, Dynamics of Multiphase Media [in Russian], Nauka, Moscow (1987).
3. V. K. Kedrinskii and R. I. Soloukhin, "Compression of a spherical gas cavity in water by a shock wave," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1961).
4. B. E. Gel'fand, S. A. Gubin, R. I. Nigmatulin, and E. I. Timofeev, "Effect of the density of a gas on the breakup of bubbles by shock waves," DokI. Akad. Nauk SSSR, 235, No. 2 (1977).
5. B. E. Gel'fand, S. A. Gubin, S. M. Kogarko, et al., "Investigation of shock-wave induced collapse of gas bubbles in a liquid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1975).
6. V. E. Dontsov, V. V. Kuznetsov, P. G. Markov, and V. E. Nakoryakov, "Evolution of a pressure wave of moderate intensity in a liquid with gas bubbles," Akust. Zh., 33, No. 6 (1987).
7. O. V. Voinov and V. V. Voinov, "Scheme of collapse of a cavitation bubble near a wall and the formation of a cumulative jet," Dokl. Akad. Nauk SSSR, 227, No. 1 (1976).

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